Interview with Henk Diepenmaat on societal innovation and prime numbers, in which he asks help from physicists and mathematicians

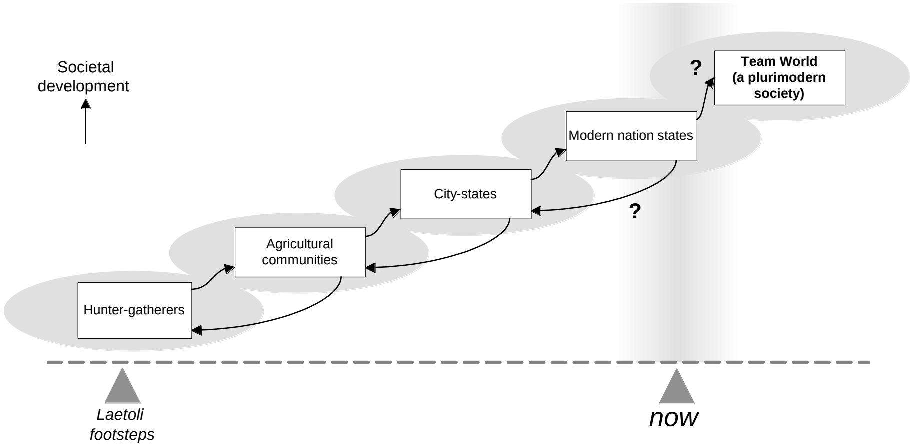
Setting the stage for

Societal entropy and a dicey proof of the Riemann Hypothesis

on the basis of **The path of humanity**: *societal innovation for the world of tomorrow*

Chris: Hi Henk, you've written a new book, *The path of humanity*. You've told me that you are quite happy with it, but at the same time something is bothering you. So tell me, what is it?

Henk: Hi Chris. Yes, it is always nice having finished a new book. But to be honest, I am in a bit of a pickle. *The path of humanity* is being published in Dutch at this very moment; in English it is due around summer. It deals with societal practices, sustainability and societal innovation. It sketches our integral human development as a bumpy non-monotonous balance seeking path from hunter-gatherers via sedentary societies, city states, and modern societies to our contemporary late modern transition episode.



The scale of this path surpasses normal experiential spheres, it defies human experience. It covers many thousands of years, and it is only during the last centuries that we are becoming aware of the presence of this path. The book explains why *The path of humanity* unfolds itself in human history and provides it with a discrete mathematical underpinning. Its central theme is that a more conscious following of our path of humanity will accelerate societal development and improve our societies. The book aims to provide methodological support in doing so.

Chris: But that sounds quite interesting and useful to me, rather than troublesome.

Henk: Yes, I agree, I think so too. And in general I am on familiar grounds with these topics. But, and here is what is bothering me: my new book uncovers a deep relationship between our integral human path of development and prime numbers. The book explores this relationship, which brought me in many unknown territories. Some of them were only new to me, but some findings are really pushing the envelope.

The path of humanity is not at all about prime numbers and number theory as such. It starts with a historical account of our human path of development so far, from hunter-gatherers to our contemporary late-modern society. It explains the many sustainability tensions of the late-modern time that we are living in. It extrapolates our path into the future, which requires a new transition.

The key breakthrough of *The path of humanity* is a philosophical stance that I call *recursive perspectivism* (I will explain it in the addendum). Recursive perspectivism provides our path of human development with a recursive element: perspectives. They possess a dual identity: they can be both situations and processes, but not simultaneously, at least according to the same consciousness. Perspectives give our path a discrete mathematical underpinning that emphasizes the *structure* of our societies and that has many connections with number theory. This is where the prime numbers come into play. After adopting this stance, perhaps hypothesis is a better word, many intriguing insights emerge. As a consequence the book features some rather remarkable and surprising entropic, statistical, relativistic and quantum theoretical characteristics of societal practices, human experience and societal development.

This may all sound rather abstract, but the book is far from merely a theoretical construct. Recursive perspectivism sheds a new and intriguing light on many phenomena, many patterns that are readily perceptible in our environment. It attempts for example to explain the fundamental reasons why empirical pattern laws like Zipf's law, Benford's law, Pareto's law and many other inversely proportional laws (both social scientific and natural scientific) are bound to describe our environment so aptly. It explains the rather contra-intuitive behavioural psychological phenomenon of *loss aversion*. It positions and explains differences between the three political mainstreams: liberalism, communism and fascism. It underpins the well-known economical *law of diminishing returns*. It substantiates why the normal (Gaussian) distribution is so abundant in our environment.

Last but not least, the book unveils and explains the path that our development is following and this both in a historical and a mathematical way. It explains the way in which human (societal) development is driven by an entropic "force" on the basis of perspectives. And as I said above: this force finds it origin in both prime numbers and perspectives. The reason for writing the book in the first place was that I surmised that this would enable us to design pragmatic methodological approaches making use of this force, and therefore supporting societal innovation. And indeed, such approaches are presented in the very last section of the book.

When intensively playing around with prime numbers, it is a matter of time before one would encounter the Riemann hypothesis. And I did. This is where it becomes really troublesome for me: the Riemann hypothesis is a complex number theoretical subject and I must confess that at times I was more than a little bit out of my depth. But I slowly came to see and understand the ways in which the Riemann hypothesis and *The path of humanity* share common grounds. *Both* rest on prime numbers, and furthermore I suspect that the way in which both develop rests on quite similar principles.

Prime numbers and the Riemann hypothesis seem to provide a structural foundation for our human nature and the long term trajectory of our societal development, our *path of humanity*. Exploring societal development sheds light on the Riemann hypothesis. Perhaps I may even have found a sort of a dicey proof for this famous hypothesis.

Chris: What are you saying Henk? Are you telling me that, as a side effect of your societal explorations, you might have proven the Riemann hypothesis, and that the proof is in your book?

Henk: With this question you touch upon the heart of the pickle. I find that question very difficult to answer. Obviously I soon became aware of the enormous standing of the Riemann hypothesis. People, far more knowledgeable about this hypothesis than I am, state that it is at the centre of deep physical mysteries. I didn't find this very reassuring, to say the least. Every sane person who is not thoroughly skilled in mathematics and physics (and I for one surely am not) would be well advised to keep away from these matters. And I admit: for this reason I approached the Riemann hypothesis rather hesitantly.

Even so, I saw interesting routes. Although the going was not easy, I followed through. At times I felt like a fish out of the water. And after a lot of hard work I think that the Riemann hypothesis is true and that it bears great relevance for societal progress. I may even have opened some new doors, and I may have made a contribution to proving it. At the very least I grounded it in human consciousness, in human development, in societal innovation. I think that is important in itself.

Chris: These are interesting statements. But math is a rather formal discipline to most of us. Either you *did* prove the hypothesis or you *didn't*. Why the beating around the bush, why the uncertainty?

Henk: I am uncertain for several good reasons.

Firstly, it is not a simple hypothesis (this is an understatement) with lots of physical relevance. I am a reflective practitioner working in multi-actor processes, rather than a disciplinary academic scholar, and I am at best an amateur in some mathematical and physical areas. The Riemann hypothesis is a highly specialized topic and stated in advanced mathematical terms. I on the other hand am far from a specialist: I do not shy away from using quite diverse assemblies of philosophical, academic and practical knowledge, covering a wide range of sciences and sectors, and I transform them at will and put them to use. To put it succinctly: although I am not *against method*, I simply gather, process, extend, integrate and use whatever works by combining and transforming it in overall pragmatic methodological frameworks.

This mode of operation is not much appreciated by more traditional disciplinary academics (and perhaps this is a euphemism). According to some of them, progress concerning the Riemann hypothesis should come from highly professional mathematicians or physicists, publishing peer-reviewed articles in highly specialized disciplinary journals of a high academic ranking, that are predominantly read by their direct peers. They may be right when aiming at highly specialized progress at specific disciplinary frontiers, and I do not doubt the relevance of this, but for societal progress I beg to differ, and in any case that's not me. I beg to differ because I think that the Riemann hypothesis is important for understanding human consciousness, human experience in general. Prime numbers are important for societal development as a whole.

Don't get me wrong: I value disciplinary academic research and I appreciate the efforts of disciplinary specialists, when performed in the right place at the right moment. Some academic researchers are truly giants, and I thankfully make use of some of their results intensively. And there should always be room for fundamental research, as there will always be use for the results of it. At the same time I think that our societies and societal innovation are in desperate need of flexible pragmatic methodological approaches that integrate many different experiential spheres. The large amounts of cultural, sectoral and disciplinary boundaries of our era, both in academia and in societal practice, are straightforwardly detrimental to further societal progress. The urgent need for sustainability and our great difficulties in achieving it are cases in point. For these reasons I think that more eclectic but at the same time more methodological roads forward should get much more attention and appreciation. Our societal practices need transitions towards sustainability, towards plurimodernity, and perhaps traditional science needs a transition in these respects as well.

Secondly, and on a more philosophical note, I consider prime numbers to be experiential phenomena, rather than formal ones. And I find it difficult to come up with reasons why it would make sense to expect that experiential phenomena can be proven completely formally.

Thirdly, my PhD thesis, dealing with intentional multi-actor modelling (University of Amsterdam, 1997), already implied a key role for prime numbers in intentional multi-actor processes and human development. Although I did not (was not able to) exploit this line of thought at that very moment, my PhD research convinced me that prime numbers are carriers of societal structure and societal development. When toiling away with discrete configurations of perspectives in my new book, I sort of stumbled on the Riemann hypothesis in a very specific interpretation: the *Denjoy* interpretation. And if this interpretation would prove to be true, I suspect that even on statistical and entropic grounds alone our *path of humanity* is most likely to converge towards societies exhibiting more and more coherence and cooperation. This would provide *The path of humanity* with an autonomous mechanism of operation, a driving force, on the basis of perspectives. This would open up routes towards intentional societal innovation, even at larger societal scales. For these reasons the Riemann hypothesis became relevant for my work, rather than that I tried to prove it from start onward. So in a sense I worked quite the other way around.

I do not want to make excessive claims. I am *not sure at all* whether I approached the Riemann hypothesis in a way that even makes the slightest sense to a professional mathematician or physicist. On the other hand, the points made above are not meant to be disclaimers or safeguards. I do not want to duck away or to take to defence. I very clearly *do* think that *The path of humanity* as a book offers some really important insights with respect to societal structure, societal development and societal innovation, on the basis of prime numbers and perspectives. This I thought through and wrote down in my new book. And I *do* think that *The path of humanity* as our common enterprise rests on the Riemann hypothesis in a very profound manner. At the risk of making a fool of myself (as Schrödinger once put it), I decided that I could do with some help from societally inclined mathematicians and physicists. And precisely this is the very reason of this interview.

Chris: I think I understand your position. You are not a specialist on prime numbers and the Riemann hypothesis. And as a non-mathematician and a non-physicist you surely wish to avoid excessive claims, cluttered by unawareness. But at the same time you state very clearly that you *did* find

something interesting, something new, something important for societal development, for societal innovation. Could you give us some clues *why* you think that you've made some significant progress?

Henk: Well, I find it difficult to summarize the reasons succinctly, so soon after the publication of the Dutch version of the book, the dust is barely settling. *The path of humanity* is a construction firmly rooted in practice and I draw upon many different philosophies and theories. At the same time it is a composition recursively built in terms of perspectives, as a colleague characterized it, rather than a gluing together of pre-existing philosophies and theories. It took me several decades to think the main concepts of the book through, and some ten years to write it down. But sure, I can shed light on some of the features of my vantage point with respect to perspectives, prime numbers and the Riemann hypothesis. I will mention four topics:

1: Recursive perspectivism and societal development,
2: Societal entropy and societal balance,
3: The Denjoy bridge between prime numbers and societal development,
4: Some highlights of the very dicey proof of the Riemann hypothesis.

It is easier to address them on paper than in an interview, so I will explain them briefly in an addendum (see Addendum 1 at the end of this transcript). I also provided a figure of the structure of the book, and a list of topics that I consider relevant for societal innovation, covered by the book. This will give you a first impression of the entire scope of the book. (See Addenda 2 and 3 at the end of this transcript.) And I added references to a presentation and a paper that I made for an English audience (see the last page).

Chris: Ok, people can read that stuff, this sheds more light on your approach. Which brings me to my last question: what do you aim for with this interview? What would you like to happen?

Henk: Well, I am hoping to find people, both educated in science on the one hand, and societal improvement and societal innovation on the other, that might throw an educated but at the same time open-minded glance at what I have unveiled in *The path of humanity*. I am hoping to find more people who understand the importance of combining both a scholarly and a practical attitude when addressing large scale societal practices.

I am thoroughly aware that the book *The path of humanity* covers a lot of ground. It may be a bit difficult to categorize. It doesn't carefully select a clear and delimited topic. Rather, the book asks whether it is possible to improve our human societies intentionally. The answer is a clear-cut and carefully substantiated "yes". And, quite surprisingly, the book explains why we *will* improve our societies on the very long run, even quite unintendedly and unaware and while going through very bad intermediate episodes, as we have done so far. An *invisible hand* _*is*_ at play, the economist and philosopher Adam Smith was right, but it surely does not make much sense to let the playing of this hand remain invisible. The book therefore sets this process right in the spotlights, and seeks ways to use it for societal innovation.

As a consequence, the subject is our integral human development, from the *Laetoli footsteps* to *our common future*. It doesn't carefully analyse this subject from a content point of view, but it rather explores, at times quite coarsely and rabidly, structural consequences of a recursive perspectivistical interpretation of human consciousness and human development.

I didn't expect the enormous thrusts forward that I experienced while working on the book. The thinking and writing was quite an enterprise; I constantly exceeded the boundaries of my knowledge, my competences and my energy. When I was ready, I looked at some 1000 pages, many of which cross the borders between philosophy, different scientific theories and practice. They freely mix alpha, beta and gamma scientific insights. Needless to say, I am far from an expert in all these matters. I am therefore prone to having made errors. The book may be somewhat of a singularity for all these reasons.

But my goal was never to be complete or completely correct. My goal was simply to uncover and better understand *The path of humanity*, because a better understanding of this path contributes to our abilities and competences in societal innovation; in achieving better, more sustainable societies. I hope this interview will spark the interests of some of the intended audience and convince them to read the book. With this interview I specifically aim at mathematicians and physicists with societal knowledge, interests and skills (both academic and practical), as I think that they are more likely to understand and appreciate the perspectivistical prime number underlayment of *The path of humanity* and the surprising entropic, statistical, relativistic and quantum theoretical consequences. And perhaps together we will be able to pull down some of the barriers, albeit very modestly, between natural and societal disciplines, between philosophies, theories and practices, between science and politics and large scale societal progress.

At this very moment the book is being published in Dutch, which explains the timing of this interview. Translation is in progress: English publication is due around the (late) summer of 2018. For the time being, I have attached some material in English focussing on prime numbers and the Riemann hypothesis (see below). It can be freely distributed. I hope some of you will read this material and the future English book, will think about and comment on the ideas, and help bringing them further. They connect societal innovation with prime numbers and, error-ridden at places that my work may be, I think continuing this work is of great importance for societal innovation, and for our human future. Intentionally using and following *The path of humanity* surely makes a lot of sense.

Chris: Thank you, Henk, for this interview.

Henk: No, Chris, thank you.

Addendum 1: A very concise first explanation of the four topics (see next page)
Addendum 2: The structure of the book *The path of humanity* in a figure
Addendum 3: A list of some topics covered by the book *The path of humanity*
Addendum 4: Additional reading

The text of this interview started as a global script for an interview between Henk Diepenmaat and his youngest son, Chris Diepenmaat. The idea was to make a video that would explain the "pickle" that Henk felt he was in after writing "The path of humanity". However, they started to edit the script, resulting in this text, and the video was never made.

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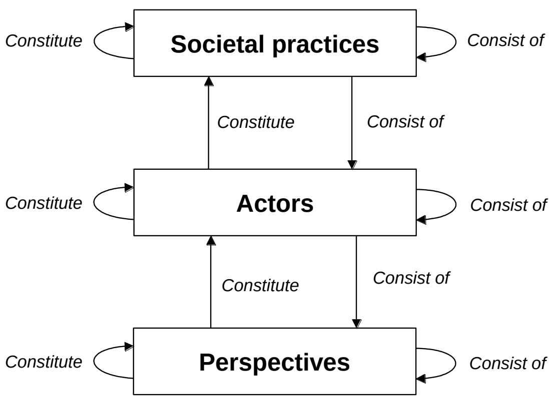
Addendum 1: A very concise first explanation of the four topics

1: Recursive perspectivism and societal development,
2: Societal entropy and societal balance,
3: The Denjoy bridge between prime numbers and societal development,
4: Some highlights of the very dicey proof of the Riemann hypothesis.

1: *Recursive perspectivism* and societal development.
For starters, *The path of humanity* is based on a specific philosophical stance I have been developing during the last decennia: ***Recursive perspectivism***. Recursive perspectivism hypothesizes that all our human experiences can be understood in terms of interconnected and recursive fluxes of *perspectives*.

A perspective simply is a triplet of an initial state, a process, and a final state. They can assume both the role of a static building block and a process, but not at the same time (a duality). In case of an *improvement* perspective, the final state is deemed better according to some actor than the initial state (some value can be, is being or has been created or preserved). In a strong version, human consciousness constitutes of, connects and processes perspectives in order to realise additional or maintain existing value. In this way Recursive perspectivism provides society with a substrate (a "stuff") that both societal practices and societal processes are made off (the aforementioned duality concept).

In a nutshell, according to recursive perspectivism, actors consist of improvement perspectives, and societies consist of actors, and this recursively. (Note that this implies that societies are multi-actor processes, consisting of perspectives.) An immense recursiveness is the result:



But then also more ambitious improvement perspectives are conceivable. For example It should be possible to let actors and complete societal practices become part of *societal* improvement perspectives, and this exactly is the reason why recursive perspectivism frames societal innovation. We are not very proficient in societal innovation yet, most of the time actors pursue their own private improvement perspectives, but the need is growing and we are learning.

2: Societal entropy and societal balance

As perspectives offer unifying building blocks (particles) of societal practices, it now becomes possible to introduce a statistical notion of a ***Societal entropy*** *that correlates with societal quality*.

Statistical entropy is known in chemistry (Gibbs' configurational entropy, for example in mixing ideal gasses) and information sciences (Shannon's information entropy, for example in coin tossing). Statistical entropic forces favour specific macrostates: they are more likely to happen, as a result of the larger number of microstates that may realise these favoured macrostates.

For example, when tossing a coin four times (p=4), for each single toss the two possible outcomes are head (H) and tail (T). The combinatorial scheme below shows all 16 possible different sequences of 4 coin tosses (from top to bottom, 16 as 2^p=16).

100	75	50	25	0	% H
H	T H H H	H T H T H T	H T T T	T	
H	H T H H	H T T H T H	T H T T	T	
H	H H T H	T H T H H T	T T H T	T	
H	H H H T	T H H T T H	T T T H	T	
0	25	50	75	100	% T
1	4	6	4	1	(group size, 4th row Pascal)

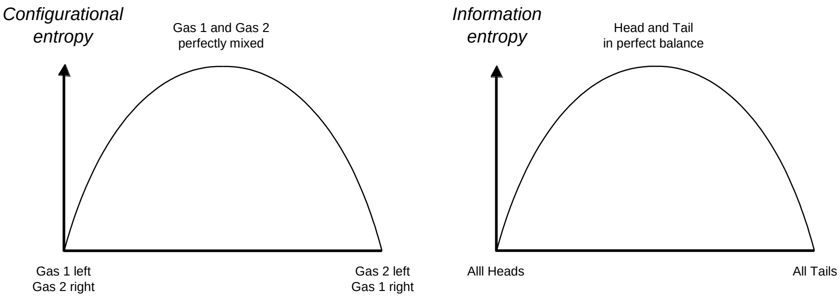
Each vertical sequence of four is a different microstate. These microstates are grouped together in macrostates on the basis of the *fractions* of H and T. The percentages of H *linearly* develop as 100% 75% 50% 25% 0% (from left to right). The percentages of T therefore develop exactly the other way around: 0% 25% 50% 75% 100%. Even in such a simple case with only 4 coin tosses, it becomes clear that – on average - tossing (50-50) is six times as easy as tossing four heads (100 – 0) or four tails (0 – 100).

A similar example is the mixing of equal amounts (let's say several moles) of two ideal gasses, A and B. The type of entropy in this case is called *Configurational entropy*. Here the number of different mixing configurations is extremely high, as many many gas particles A and B are at play.

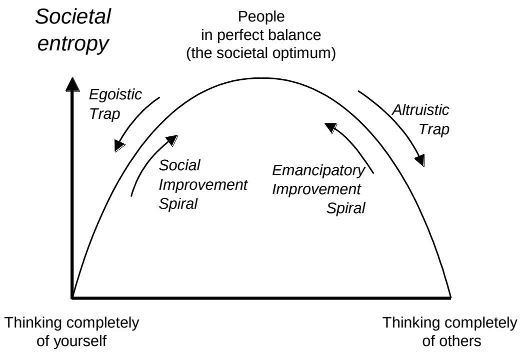
In case of tossing a coin with two sides four times, the number of microstates per macrostate will nicely follow the symmetrical 4th row (the sequence is 4) of Pascal's binomial

Triangle: 1 4 6 4 1. For longer sequences, this binomial pattern of the higher and higher corresponding row of Pascal's Triangle will resemble the Gaussian or normal curve, well known from statistics, better and better. As a result, repeating a sequence of p coin tosses over and over on average will result in a macrostate distribution that will converge towards the p^{th} row of Pascal's Triangle.

In case of many configurations, and under the assumption that each microstate is equally probable, the entropy equals the natural log (ln) of the number of microstates that may realize a macrostate (times a factor). For binomial distributions that follow high rows of Pascal's Triangle, like long sequences of coin tosses, or mixing two equal volumes of ideal gasses, this results in a very typical curve: the entropy curve (see below). These statistical entropy curves result from taking the natural log (ln) of the number of possibilities (microstates) at each position (macrostate) on the x-axis. Given enough building blocks, the number of possibilities follows a high row of Pascal's Triangle, and after taking the natural log a very characteristic entropy curve like this will be the result:



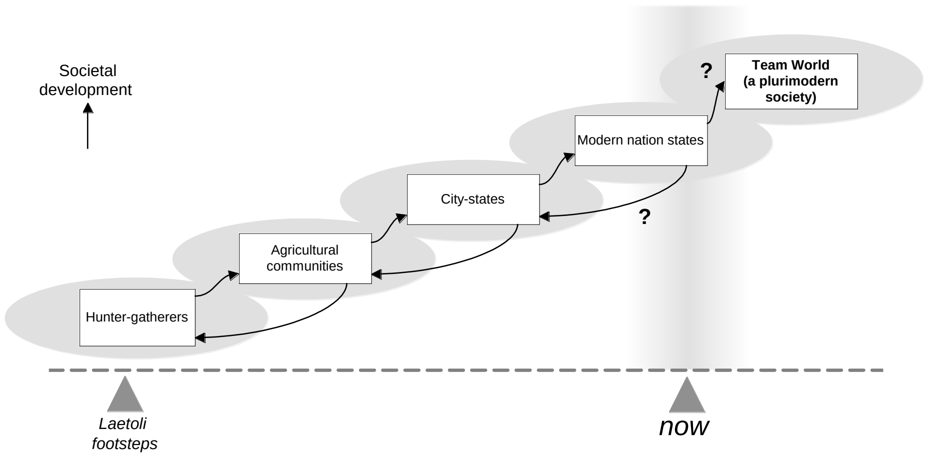
Recursive perspectivism enables applying this type of reasoning on societal structure as well, resulting in a *Societal entropy*. We may for example distinguish two different types of improvement perspectives in your experiential sphere: the ones favouring *yourself*, and the ones favouring *others*. The result will be the well-known entropy curve:



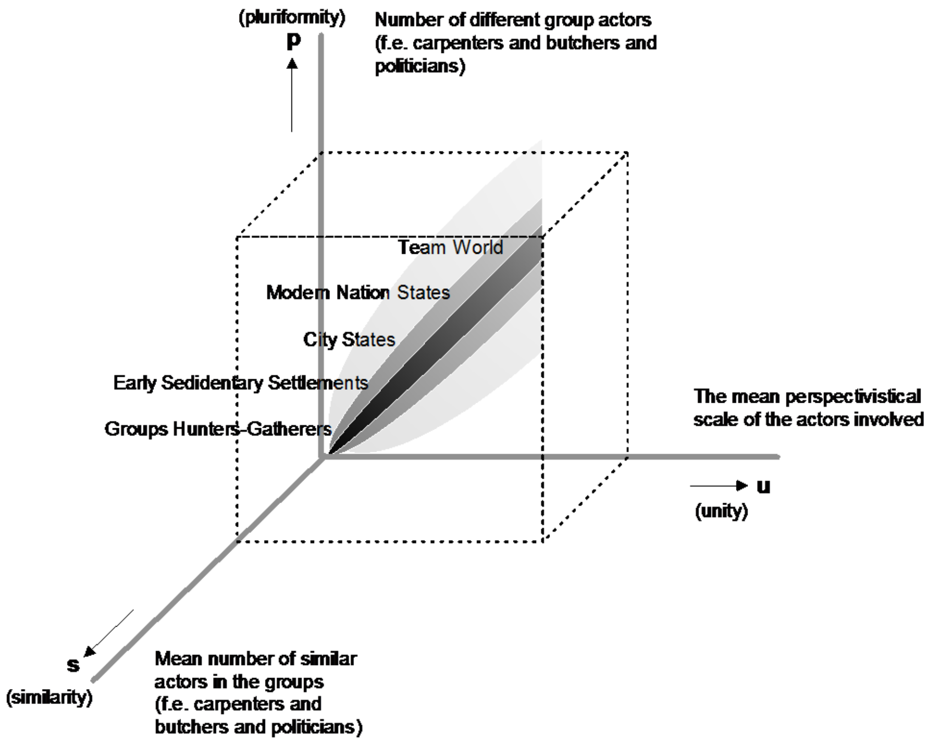
The societal balance model (see "*The path of humanity*")

As a consequence of this societal entropy, societal practices will seek their optimum half way, in much the same way that a longer and longer sequence of *coin tosses* will seek a balance half way, and two equal amounts (particles) of different ideal gasses will mix more and more perfectly.

This introduces the notion of a **societal balance**. *The path of humanity* is the bumpy entropy driven and balance seeking path we humans have walked from hunter-gatherers via sedentary societies, city states, and modern societies to our contemporary late modern societies in transition. It is fair to say that most of us live our lives quite unaware of this path and this process, even in complete oblivion. The scale of the path defies human experience, as it surpasses normal experiential spheres by far. It is only during the last centuries that some of us are becoming aware of the presence of this path.

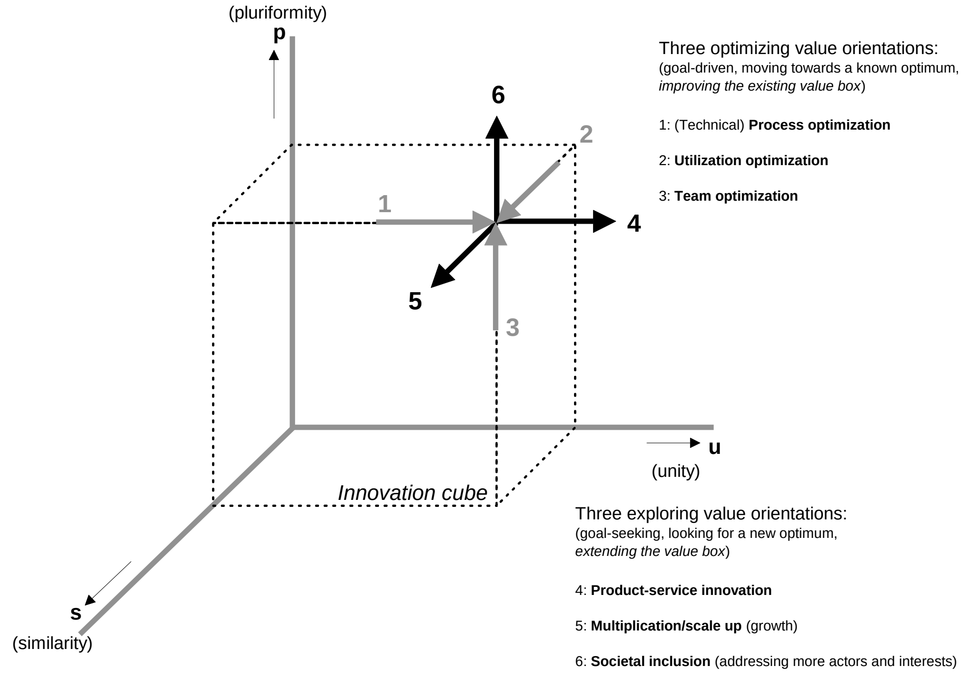


The book explains why *The path of humanity* unfolds itself in human history even while we humans are unaware of the presence of this path in our dayly decisions: the underlying upward societal force is of a purely entropic nature. There simply are far far more possible societal (perspectivistical) configurations half way than near the sides. The book applies the same arguments in three perspectivistical dimensions as well, positioning *The path of* *humanity* on (near) the diagonal (I will not explain the dimensions here, but the overall number of perspectives at stake equals **p**.**s**.**u**):



Recursive perspectivism helps us in understanding that, given enough time and a certain amount of perspectives, the structure of a society will develop itself spontaneously towards the point of balance (i.e. the societal entropy increases). This is far from a monotonous process, see the coin tossing analogue, but on the very long run and in very complex situations this process of loose convergence will emerge in any case. This resembles Adam Smith's much discussed *invisible hand*. On the very long run and in very complex situations involving enormous amounts of microstates, this entropic process boils down to the Second Law of Thermodynamics, which states that the total entropy of an isolated system can never decrease over time. From a microscopic and statistical point of view this is a rather sloppy formulation, by the way, as many short term very small decreases are to be expected. Also larger scale decrease will manifest itself, albeit far more seldom.

The key message of the book "The path of humanity" is that we may accelerate processes of societal innovation dramatically, by becoming aware of these entropic forces and steering our societal practices towards higher levels along the path of humanity. We simply should start doing consciously, intentionally and professionally what history tells us we've done statistically and rather unaware for thousands of years before. In other words: we turn the *invisible hand*, operating quite unawarely, into a *visible hand*, steering intentionally. For this purpose, the book provides several methods (this is the topic of Section 3 of the book: Societal innovation.) The figure below presents one of them: the **Innovation Cube**, which makes this key message operational by distinguishing six different innovation methods (value orientations) that should be used in flexible combination in order to be able to steer diagonally towards better societies (the six methods are explained in section 3: Societal Innovation, of *The path of humanity*).



Value orientation\ *Focus*	*Actors*	*Technology*	*Resources*	*Optimization*	*Exploration*
1: Process optimization	weak	strong	weak	strong	weak
2: Utilization optimization	weak	weak	strong	strong	weak
3: Team optimization	strong	weak	weak	strong	weak
4: Product-service innovation	weak	strong	weak	weak	strong
5: Multiplication/scale up	weak	weak	strong	weak	strong
6: Societal inclusion	strong	weak	weak	weak	strong

3: The Denjoy bridge.

A societal practice is a configuration in a multi-dimensional perspectivistical space, and possible configurations are governed by primes. Recursive perspectivism enables us to relate societal development to coin tossing on combinatorial and entropic grounds. But also prime numbers and the Riemann hypothesis are related to coin tossing. Coin tossing therefore provides a bridge between societal development and the Riemann hypothesis, as I will explain below.

The Riemann hypothesis is named after Bernhard Riemann, the German mathematician, physician and philosopher (1826-1866) who rather casually mentioned it, and is generally stated in a complex number vocabulary:

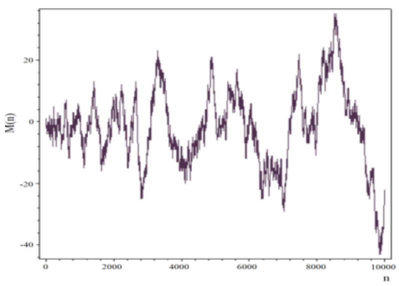
"All the nontrivial zeroes of the analytic continuation of the Riemann zeta function ζ have a real part equal to ½."

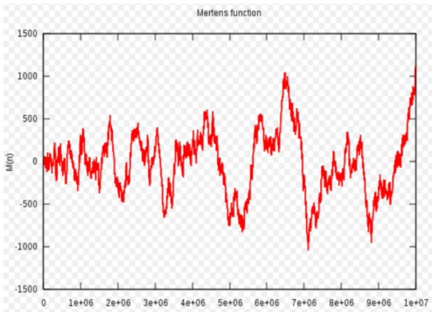
This hypothesis is deeply connected to the *Mertens* function (named after the German mathematician Franz Mertens, 1840-1927), a function in its turn built on the *Möbius* function and dealing with positive whole numbers (the realm of recursive perspectivism). The Möbius function μ(n) (named after the German mathematician August Ferninand Möbius, 1790-1868) equals 1 for n=1, 0 for squared numbers (a squared number has at least one double prime factor), -1 for square-free numbers with an odd number of prime factors, and +1 for square-free numbers with an even number of prime factors.

The Mertens function M(m) simply is the sum of the Möbius values (I will call this a "Sigma Möbius", see further on) from 1 to m:

$$M(m) = \sum_{n=1}^{m} \mu(n)$$

The Mertens function is shown below for m from 1 to 10.000 (left, source: Wikipedia) and for m from 1 to 10.000.000 (right, source: Wikimedia). Many people see merely noise. *I however see a specifically interleaved part of a binomial bell curve.*





Edwards (Riemann's zeta function, 1974, paragraph 12.1), following Littlewood, provides this connection by means of a direct equivalent of the Riemann hypothesis in terms of the Mertens function:

"If $M(x) = O(x^{(½ + ε)})$ is true with probability one, the Riemann hypothesis is true with probability one."

In order to prove the Riemann hypothesis, it would therefore suffice to prove that M(x) grows less rapidly than $x^{(1/2 + ε)}$ for all ε > 0 (see Edwards paragraph 12.1).

As a result of the above investigating the similarities between the Mertens function and a coin tossing sequence (typically referred to as a Bernoulli experiment in mathematics) offers an intriguing possible pathway to proving the Riemann hypothesis. This is explained in Edwards (1974, paragraph 12.3: *Denjoy's probabilistic interpretation of the Riemann* *hypothesis*), and I will follow this paragraph. In a Bernoulli experiment (for example a coin tossing sequence):

> *"with probability 1 the number of Heads minus the number of Tails grows less rapidly than* $N^{(1/2+\varepsilon)}$*."*

This is because of two reasons: 1) the probability of a Head equals the probability of a Tail and 2) the occurrence of Heads and Tails is independent of each other.

Edwards then argues that it is not altogether unreasonable to assume that in the Mertens function the occurrence of μ(n) = +1 equals the occurrence of μ(n) = -1, and that occurrences of +1 and -1 are independent of each other. If, however, these two assumptions would apply, the conclusion would be that M(x) behaves exactly the same as a Bernoulli experiment. The equivalent statement of the Riemann hypothesis in terms of the Mertens function, at the last page, would then be true. *Prove these two not unreasonable assumptions, and you will have proven the Riemann hypothesis*. This is called **Denjoy's probabilistic interpretation** of the Riemann hypothesis (after the French mathematician Arnaud Denjoy).

The Denjoy pathway, however, is dicey. After some fruitless attempts I decided that the Mertens function and a Bernoulli experiment are quite different, the book elaborates on this. The Mertens function on the one hand is completely determined: its graph will be the same over and over. In contrast to this, and on the other hand, each coin tossing sequence will show its own stochastic pathway, asymptotically bound by $N^{(1/2+\varepsilon)}$ but resulting in quite a unique graph. Understanding their similarities is difficult. The Denjoy pathway is dicey indeed.

Different they may be, but prime numbers exhibit remarkable combinatorial patterns (bottom-up), which make it possible to distil binomial patterns from the whole number line (bottom-down). Again, I will not dwell on this, the book, the paper and the presentation will elaborate, but I will show some intriguing results without further comments, to give you a first impression.

Primorial numbers are the product of the first n prime numbers. For example: 2, 6=2.3, 30=2.3.5, 210=2.3.5.7 et cetera. Adding the next prime number I call a primorial step. Because of the square-free nature of primorial numbers (duplicate prime factors are completely absent) we can use the combinatorial function for the binomial coefficients (Newton's binomial theorem) to calculate the number of factors of a primorial number consisting of k of the p prime factors. If any set has p *different* elements, the number of different combinations for each k is equal to the *binomial coefficient*:

Comb (p, k) = p! **/** k!(p-k)!

Take for example a set of three different fruits: an apple, a banana and an orange. The possibilities to select different sets of 2 are: (apple banana), (apple, orange), (banana, orange), so three different sets. This number of different sets of two out of these three fruits can be calculated using the combinatorial formula:

Comb (3, 2) = 3! **/** 2!1! = 3

In order to show the way in which this is intimately related to Pascal's Triangle and the binomial bell curve, I use a function $\mathbf{f}_p$. This function calculates the number of all possibilities consisting of k elements, k going from 0 to p, and adds them together:

$\mathbf{f}_p$ =

Comb (p, 0) +	(k=0)
Comb (p, 1) +	(k=1)
Comb (p, 2) +	(k=2)
… +	(k=..)
Comb (p, p)	(k=p)

More concisely:

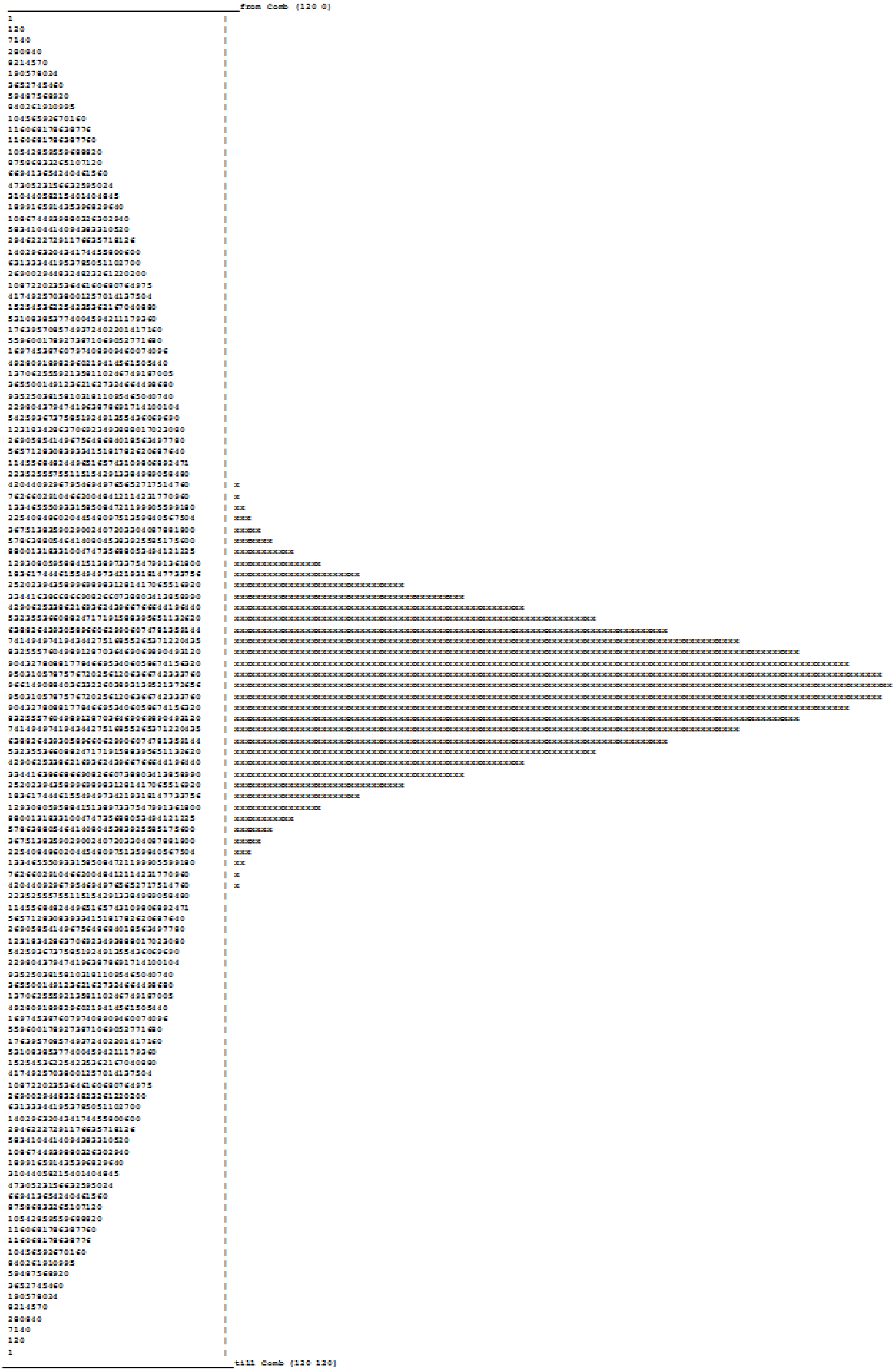
$$\mathbf{f}_p = \Sigma k_{0\text{->}p} \text{ Comb } (p, k)$$

Comb (p, 0) results in 1: one possibility of taking none of the set members exists. Similarly Comb (p, p) will result in 1: there is one way to include all the elements of an unordered set.

$\mathbf{f}_p$ is convenient, as writing out all the subsets is rather cumbersome when p is large. Actually this is an understatement: the number of subsets becomes astonishing for large p's. See the p=120 example on the next page: the added result (the sum value) is 2^{120}= 1329227995784915872903807060280344576, an enormous amount. It doesn't matter whether it concerns 120 types of fruit, 120 types of cars, *or 120 different prime numbers*. The p members constituting the set must be different, in the sense of being distinguishable from each other in the Boltzmannian macro sense. Under this condition he resulting pattern will follow the p-th row of Pascal's triangle (the left part of the large picture below is calculated by means of $\mathbf{f}_p$ for p=120, and therefore results in the 120-th row of Pascal's triangle), and the result will be a binomial bell curve (see the large picture on the next page).

When using $\mathbf{f}_p$ for combining prime factors of a primorial number, multiplication of the resulting sets of prime factors will result in the *factors* of this primorial number. Primorial steps double the number of factors, this number equals 2^p and each step increases p by 1. The new factors resulting from a primorial step will be added, and are *completely scale invariant* with respect to the already existing factors of the former step, as the new factors simply are the old factors multiplied by the newly added prime factor. This also implies that already existing factors with an odd number of prime factors will be 1-1 accompanied by factors with an even prime factorization, vice versa.

As a consequence, the ratio between factors with an odd and an even prime factorization will remain exactly 50%-50%. For example: stepping from 6=(2 3) to 30=(2 3 5) extends the four factors (1 2 3 6) of 6 with the four factors (5 10 15 30), their value is exactly five times (the newly added prime factor) the existing ones, resulting in eight factors: (1 2 3 5 6 10 15 30). The old factors in terms of their number of prime factors were: odd, odd, odd, even; whereas the new factors are: odd, even, even, even, resulting in an equal amount again.




```
top value  96614908840363322603893139521372656
sum value  2^120=1329227995784915872903807060280344576
one x is   805124247223947276916044561645568
sum/top    13.758
```

The *discrete inversely proportional relationship* of the new primorial number n=x.y will provide the necessary and sufficient whole number positions for these factors: (1 30)(2 15)(3 10)(5 6)(6 5)(10 3)(15 2)(30 1). The book interprets this phenomenon as a quantum theoretical effect, with some rather surprizing consequences for societal development and likely societal structures, but this is off topic here.

Primorial steps change factors *both* in terms of distribution over k ranges (binomial structure) *and* in terms of relative order (position) on the whole number line (numerical content). See the three consecutive primorial numbers below. Stepping up is from 210 to 2310 to 30300, stepping down is the other way around. The new factors due to primorial steps down are in italics and underscored. Note that while adding or deleting factors, the steps nicely obey the rows of Pascal's triangle in binomial, structural terms. Also note the symmetries exhibited in these structural patterns of factors (they are rather hidden on the number line due to *interleaving*, the presence of squared numbers and the presence of "strange" square-free numbers, see the paper, the presentation and the book).

```
210=(2.3.5.7) p=4

   k=0 =>  1     (1)
   k=1 =>  4     (2 3 5 7)
   k=2 =>  6     (6 10 14 15 21 35)
   k=3 =>  4     (30 42 70 105)
   k=4 =>  1     (210)


   2310=(2.3.5.7.11) p=5

   k=0 =>  1     (1)
   k=1 =>  5     (2 3 5 7 11)                                       up: +1    down: -1
   k=2 =>  10    (6 10 14 15 21 22 33 35 55 77)                     up: +4    down: -5
   k=3 =>  10    (30 42 66 70 105 110 154 165 231 385)              up: +6    down: -10
   k=4 =>  5     (210 330 462 770 1155)                             up: +4    down: -10
   k=5 =>  1     (2310)                                             up: +1    down: -5
                                                                              down: -1

   30030=(2.3.5.7.11.13) p=6

   k=0 =>  1     (1)
   k=1 =>  6     (2 3 5 7 11 13)
   k=2 =>  15    (6 10 14 15 21 22 26 33 35 39 55 65 77 91 143)
   k=3 =>  20    (30 42 66 70 78 105 110 130 154 165 182 195 231 273 286 385 429 455 715 1001)
   k=4 =>  15    (210 330 390 462 546 770 858 910 1155 1365 1430 2002 2145 3003 5005)
   k=5 =>  6     (2310 2730 4290 6006 10010 15015)
   k=6 =>  1     (30030)
```

Now consider the following five p=4 square-free example numbers (the fifth is the p=4 primorial number, 210):

```
   8756100193        =(293 307 311 313)
   46189             =(11 13 17 19)
   1938              =(2 3 17 19)
   462               =(2 3 7 11)
   210               =(2 3 5 7) (the p=4 primorial number)
```

The number of factors must be the same in all cases: $2^4=16$, as they are all square-free p=4 numbers. Their binomial structures should therefore be exactly the same as well (as of course they are). Their *interleaving* however is different. The level of interleaving is a complex stepwise process, depending on the relative size (the relative order of magnitude) of the prime factors of a number of concern, as all factors are *products* of these prime factors. Interleaving therefore is a typical content related binomial effect, belonging to *numbers* (f.e. *fruits* do not interleave).

The relative order of magnitude of the prime factors of the five example numbers is quite different. If we would order their factors according to the number line, and only *after that* draw the Sigma Möbius of these factors, we would take into account the interleaving. The x-axis now is ordered according to the number line. If interleaving is present, the resulting Sawtooth will readily show this as a deviation, an exchange of places with respect to the ideal binomial Sawtooth. The resulting graphs I therefore call *Ordered Sawtooths*, as in an Ordered Sawtooth the factors are interleaved if required. They are ordered according to the number line. This in contrast with the normal or binomial Sawtooths, these simply and blindly follow the binomial pattern of the row of Pascal of concern.

Interleaving cannot shorten a Sawtooth (as squares do): factors are *changing place*, rather than *vanishing*[1]. This may seriously destroy the sharp teeth of the "ideal" binomial Sawtooth pattern according to the rows of Pascal's triangle, as especially the points of the Sawtooth teeth are most prone to interleave: they represent the beginnings and ends of the different k-ranges. The Ordered Sawtooths of the five p=4 square-free numbers are presented below, at right is the p=4 primorial number, 210.

```
8756100193=(293 307 311 313)

fact       |   |
1          |   |X
293        |   X
307        |  X|
311        | XX|
313        |XXX|
89951      | XX|
91123      |  X|
91709      |   X
95477      |   |X
96091      |   |XX
97343      |   |XXX
27974761   |   |XX
28154663   |   |X
28521499   |   X
29884301   |  X|
8756100193 |   X
```

```
46189=(11 13 17 19)

fact       |   |
1          |   |X
11         |   X
13         |  X|
17         | XX|
19         |XXX|
143        | XX|
187        |  X|
209        |   X
221        |   |X
247        |   |XX
323        |   |XXX
2431       |   |XX
2717       |   |X
3553       |   X
4199       |  X|
46189      |   X
```

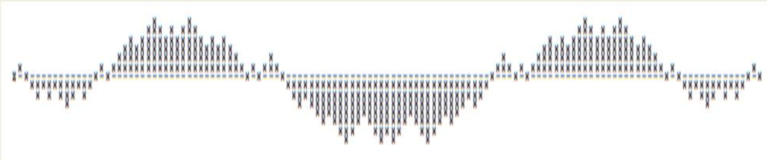
```
The p=4 primorial
210=(2 3 5 7)

fact       |   |
1          |   |X
2          |   X
3          |  X|
5          | XX|
6          |  X|
7          | XX|
10         |  X|
14         |   X
15         |   |X
21         |   |XX
30         |   |X
35         |   |XX
42         |   |X
70         |   X
105        |  X|
210        |   X
```

```
1938=(2 3 17 19)

fact       |   |
1          |   |X
2          |   X
3          |  X|
6          |   X
17         |  X|
19         | XX|
34         |  X|
38         |   X
51         |   |X
57         |   |XX
102        |   |X
114        |   X
323        |   |X
646        |   X
969        |  X|
1938       |   X
```

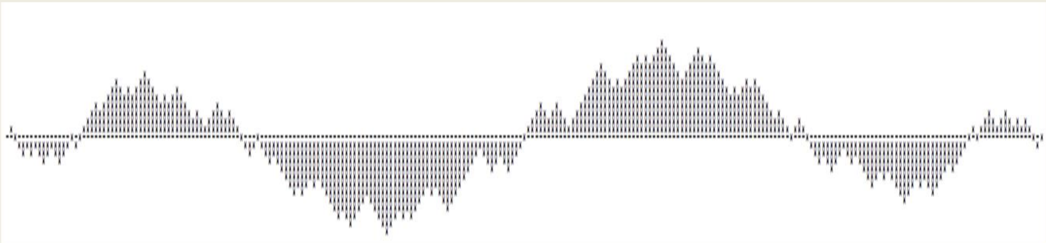
```
462=(2 3 7 11)

fact       |   |
1          |   |X
2          |   X
3          |  X|
6          |   X
7          |  X|
11         | XX|
14         |  X|
21         |   X
22         |   |X
33         |   |XX
42         |   |X
66         |   X
77         |   |X
154        |   X
231        |  X|
462        |   X
```

[1] See "emerging and vanishing properties" in my thesis.

When looking at the Ordered Sawtooth of the p=4 primorial, 210 (the figure above at the right), you might see the emergence of a very typical pattern, well known in quantum physics: a wave pattern. For the p=4 primorial this might not be very convincing yet, but the Ordered Sawtooths of higher primorial numbers like p=7 and p=8 readily reveal their secret (7 and 8 teeth are present respectively, when neglecting the much smaller “in between” teeth):



510510=(2 3 5 7 11 13 17) p=7 2^7 = 128 factors



9699690=(2 3 5 7 11 13 17 19) p=8 2^8 = 256 factors

Note that the basic Sawtooth patterns (the binomial patterns without interleaving) shape these wave patterns (as a consequence the symmetry is different for even and odd numbers of prime factors), and the interleaving turns them into the Ordered Sawtooth wave patterns. The highest peaks (teeth) of the Sawtooth patterns of primorial numbers are most heavily replaced when ordering them according to the number line: they consist of the highest lower k factors and the lowest higher k factors, and therefore they are most likely to interleave. When making a primorial step, teeth interleave within their own boundaries, it may be compared with the eroding of a sand castle at the beach, and this tantalizing process results in the typical quantum waves. Indeed they rest on a quantum process, as factors are only allowed to fill the inversely proportional line n=x.y with *whole* number x and y values (perspectives act like quants). When making primorial steps, f.e. from 6=(2 3) to 30=(2 3 5) to 210=(2 3 5 7) to 2310=(2 3 5 7 11), all the non-primorial numbers in between the numbers of the primorial sequence will obey and fill *their* specific non-primorial formula n=x.y with factors as well.

4: Some highlights of the very dicey proof of the Riemann hypothesis

A crucial key to understanding the relevance of primorial steps and interleaving for proving the Riemann hypothesis is the definition (perhaps the distinction) of *three categories* of numbers *from 1 to a (any) primorial number*:

1. *Factors* of this primorial number (square-free, see the sigma Möbius wave pattern)
2. Squared numbers (they are Möbius 0)
3. Square-free non-factors

We can establish two things now:

1: The Sigma Möbius over the cat 1 numbers (the factors) of a primorial number is 0. For primorial numbers with an even prime factorization, the Sigma Möbius over half the cat 1 numbers is 0.

2: The Sigma Möbius over *the cat 1 numbers of a growing primorial sequence* will grow less rapidly than $x^{(1/2+\varepsilon)}$ for all ε > 0.

However, this does not suffice for our purpose: proving the Riemann hypothesis. In order to do this, the Sigma Möbius over *all* the numbers (which is the Mertens function) should grow less rapidly than $x^{(1/2+\varepsilon)}$ for all ε > 0.

Cat 2 numbers cannot be turned into cat 1 or cat 3 numbers: they simply are what they are. This identifies cat 3 numbers as the *proof spoilers* of the Riemann hypothesis: they prevent the establishment that M(x) grows less rapidly than $x^{(1/2+\varepsilon)}$ for all ε > 0. They also prevent the Mertens function from 1 to a primorial number (the Sigma Möbius from 1 to a primorial number) from becoming 0 (which essentially is the same).

See for example the primorial segment from 1 to 30=(2 3 5), below. On the second row, the three categories of the numbers are specified. On the third row, the value of the Mertens function is provided.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1	1	2	1	1	3	2	2	1	3	2	3	3	1	2	3	2	3	2	3	3	3	2	2	3	2	2	3	1
1	0	-1	-1	-2	-1	-2	-2	-2	-1	-2	-2	-3	-2	-1	-1	-2	-2	-3	-3	-2	-1	-2	-2	-2	-1	-1	-1	-2	-3

The value -3 of M(30) at the right end of the third row (the Mertens row) is *completely due* to the category 3 numbers (the proof spoilers), as the sigma Möbius over cat 1 numbers (the factors) of a primorial number equals 0. (The Möbius value of cat 2 numbers, the squared numbers, is 0.)

The crucial step in proving the Riemann hypothesis therefore is dealing with the proof spoilers: the cat 3 numbers. And one way of doing this is getting rid of them.

From the point of view of the Riemann hypothesis, it is very interesting to know the way in which the fractions of cat 1 and cat 3 numbers will develop on the whole number line during continual primorial steps. If during the development of the primorial sequence cat 1 would systematically and progressively drive out cat 3, the proof spoilers would disappear, $M(x) = O(x^{(1/2+ \varepsilon)})$ would become true with probability one, and therefore the conclusion would have to be that the Riemann hypothesis is true with probability 1.

And this is exactly what happens (it is explained in the paper, the presentation and the book). While making primorial steps, the condensation area, an increasingly and divergently growing segment from 1 upward, will not contain *any category 3 numbers whatsoever*. Simultaneously, and for symmetrical reasons, the free zone (free of cat 1 numbers) will develop between the *consecutive* primorial numbers and their halves. A stronger and stronger microscope will be required to be able to notice the condensation area, as it becomes very (very! very!!) small with respect to the growing primorial segment. Nonetheless, the condensation area is a progressively upward moving frontier, and divergent. The consequence of this is that, when looking at Mertens function, starting at 1 and ending at m, no matter how big m, we are always looking at a cat 3 free part of an enormous (enormous!) primorial number (and any primorial number bigger than that one). We got rid of the proof spoilers.

As a result, $M(x) = O(x^{(1/2+ \varepsilon)})$ is true with probability one, and therefore the Riemann hypothesis should be true with probability 1.

End of explanation of the four topics.
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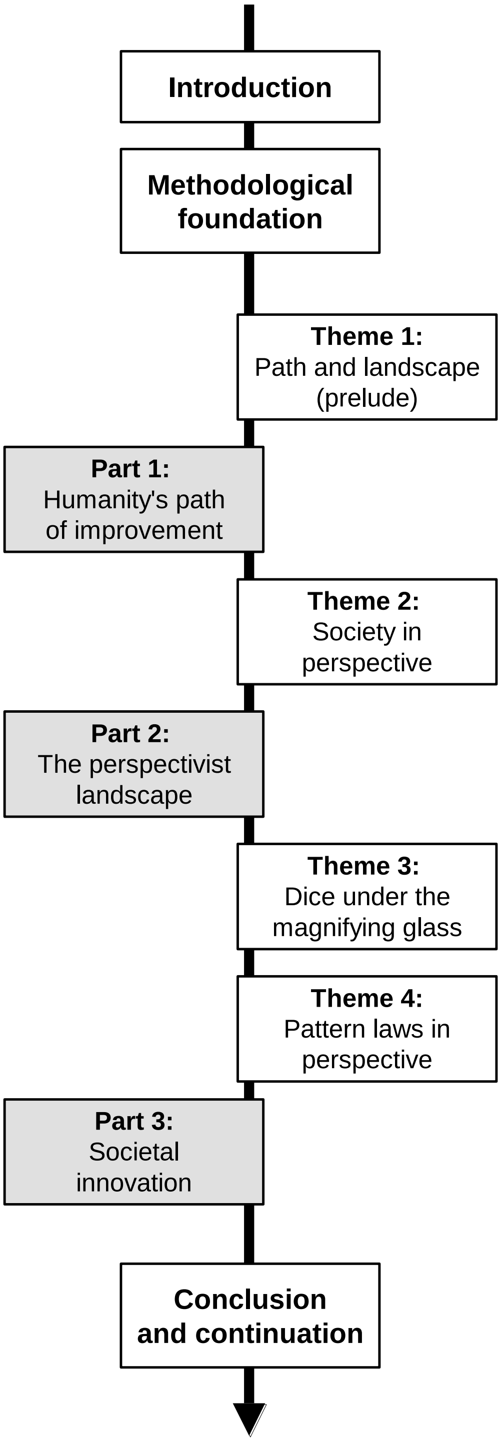
Addendum 2: The main structure of the book *The path of humanity* in a figure



The entire line of the book
"The path of humanity: societal innovation for the world of tomorrow"

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Addendum 3: A list of topics covered by The path of humanity

The list below freely sums up some of the topics, more or less in order, to give a first impression.

1. A path of humanity emerges quite unintentionally, and following it consciously makes sense
2. Description of The path of humanity from a historical point of view (content)
3. A methodological framework for societal improvement
4. Subject-object and mind-matter: from duality to a spectral world
5. (Cognitive) dissonance, tension, improvement and intentionality
6. Perspectives and recursive perspectivism: the substrate of human and societal experience
7. Consciousness and perspectives
8. Perspectives on double track: contents and structure; history and quantum theory
9. Introduction of the path of humanity in whole number space (structure)
10. Prime numbers determine societal structures
11. Continuous illusions in a discrete world
12. Why continual incrementalism is impossible in societal progress (the bumpy road)
13. Good and bad definitions of teams; Team World
14. Why, in general, societal teams are good and societal chains are not good
15. The swan song of modernity: jumping towards the plurimodernity of Team World
16. Societal practices, actors and perspectives: a highly recursively model
17. Society as a multi-actor process: the radical actor thesis
18. Intentional logics
19. Intentional multi-actor models
20. A Copernican turn from dualism to a spectral plurimodernity
21. Time and space as emerging properties of recursive perspectivism
22. Entropic analogies and entropic balance: configurational entropy (gasses), information entropy (coin tosses), and societal entropy (societal practices; societal balance)
23. The societal balance model
24. Exposing the invisible hand
25. Three societal dimensions: pluriformity, similarity and unity, and our diagonal path
26. Societal balance and relativism: why extreme political ideologies like neoliberalism, fascism and communism do not cut to the chase on the long term
27. Beyond entropy: tritropy explains cooperation and coherence in societal practice
28. On harmony: value potential, value experience and a harmonic explanation of loss aversion
29. Societal configurations and improvement potentials
30. The discrete inversely proportional relationship and explanations of Zipf's, Benford's and Pareto's laws
31. A societal quantum theory explaining scales and levels
32. The Riemann hypothesis: a dicey proof
33. Societal innovation amounts to carefully following The path of humanity
34. Making progress towards a sustainable future: three stages in sustainable development
35. The Penta Helix as a foundation for societal innovation
36. The value paradox: continuing your value path during innovation may be *SMART* but is not wise
37. Turning the value paradox into a societal value orientation: the statistical double hypothesis
38. Societal innovation requires mixing six innovation methods: the societal innovation cube
39. The backbone of societal innovation
40. Proficiency levels in societal innovation

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Addendum 4: Additional reading

The following English sources can be obtained via henk.diepenmaat@actors.nl or www.henkdiepenmaat.nl/path:

1: The back cover summary, some endorsements and a short biography

2: This interview

Two sources, V-C and V-D, addressing the relationships between societal innovation and the Riemann hypothesis. They describe the dicey proof further. It is advised to read the interview before reading these two sources, as it explains their setting.

3: V-C, PowerPoint Presentation
Relations between societal innovation and number theory: **Dicey proofs of the Riemann hypothesis** on the basis of the forthcoming book **The path of humanity:** societal innovation for the world of tomorrow, Henk Diepenmaat, 2017-2018

4: V-D, Paper
A dicey proof of the Riemann hypothesis inspired by societal innovation,
Henk Diepenmaat, 2017-2018

The main source is the book itself:

Dutch: Henk Diepenmaat, "**Het pad van de mensheid**: maatschappelijke innovatie voor de samenleving van morgen", Samenleving in Perspectief, Deel V, Uitgeverij Parthenon, Almere, 2018 (paper version, € 39,90 and e-book version, € 5,-). A preview in Dutch: www.henkdiepenmaat.nl/pad

English: Henk Diepenmaat, "**The path of humanity**: societal innovation for the world of tomorrow"
Society in perspective, Part V, Parthenon Publishing House, Almere, 2018
(in translation, expected summer 2018)

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